If we eliminate the a's from equations (3) we obtain

$$\begin{vmatrix} -1 & \cos(a_1a_2) & \cos(a_1a_3) & \cdots & \cos(a_1a_n) \\ \cos(a_1a_2) & -1 & \cdots\cdots & \cdots & \cos(a_2a_n) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cos(a_1a_n) & \cdot & \cdot & \cdot & -1 \end{vmatrix} = 0 \tag{4}$$

as an identical relation between the cosines of the angles. For a quadrilateral (4) becomes, if we denote $\cos(a_ia_j)$ by (i, j),

$$1 - \Sigma(ij)^2 + [(12)(34) - (13)(24) - (14)(23)]^2 - 2(12)[(13)(23) + (14)(24)] - 2(34)[(13)(14) + (23)(24)] = 0.$$

UNDERGRADUATE MATHEMATICS CLUBS.

EDITED BY R. C. ARCHIBALD, Brown University, Providence, R. I.

CLUB ACTIVITIES.

THE MATHEMATICS CLUB, University of Colorado, Boulder, Colo.

This club was organized in October, 1915, "to stimulate interest in mathematics among those who have had calculus." The total membership this year is 41 and the average attendance about 30. Professor George H. Light acts as chairman of the meetings and the following program for 1917–18 was arranged by him "with the assistance of club members," and issued in printed form.

November 20: "Non-Euclidean Geometry" by Leroy A. MacColl '19;
December 4: "Discovery of Logarithms" by Leona E. Vincent '19;
December 18: "Squaring the Hyperbola" by Ada G. Hall '18; "Probability in Arithmetic" by Henry A. Howell '18;
January 15: "Condition that $f(x, y, z)$ can be factored" by Agnes M. Wright '20;
February 5: "Applications for Vectors" by Claribell Kendall, instructor in mathematics;
February 19: "Nth Dimensions" by Lauren C. Hand '19;
March 5: "Relativity in Astronomy" by Edgar W. Wollard '20;
March 19: "American Mathematicians" by Dorothy Bair '20, and Alfreda Alenius '21;
April 2: "Proofs of Pythagoras's Theorem" by Lila Nelson '20; "Geometric Proof that $\sin 3A = 3\sin A - 4\sin^3 A$" by Oliver De Motte Sp.;
April 16: "Certain Definite Integrals" by Mildred McMillen '19;
May 7: "Curve Tracing" by Anthony J. Killgore '20;
May 21: "Famous Problems in Mathematics" by Gussie Wellman '21.

THE MATHEMATICS CLUB, Harvard University, Cambridge, Mass.

At least as far back as 1898 there flourished at Harvard an organization known as The Mathematical Conference, in which all students pursuing advanced studies in mathematics were invited to take part. The conference was "intended for the presentation and discussion of work done in courses of reading and research, of articles in the mathematical journals and of other suitable matter, and for the meeting of instructors and students." The meetings were held twice a month. In 1902 it was announced that "the direction of the Conference is in the hands of a committee of graduate and undergraduate students, acting with the advice and assistance of the Division of Mathematics." In the following year the number of meetings was reduced to one a month and it was officially published that "pains will be taken to provide for the discussion of a fair proportion of subjects within the capacity of members of the younger undergraduate classes. An important object aimed at is the free and general interchange of views between students and instructors. . . . Students of Radcliffe College are cordially asked to be present and to contribute to the discussions."

In the autumn of 1904 the Mathematical Conference resolved itself into The Mathematical Club, of which the first officers were: President, William H. Roever; secretary and treasurer, Ralph B. Stone. These officers together with Dr. Julian L. Coolidge, instructor in mathematics, constituted the executive committee. The guiding principles now are as when reorganized thirteen years ago. "Meetings are held once a fortnight, in the evenings, and simple refreshments are served after the more formal part of the meetings. Any student who has taken or is taking a course in mathematics not regularly open to freshmen is eligible for membership in the club; and a special effort is made to have a good proportion of papers which shall be of interest to students taking courses of intermediate grade. A small membership fee is charged[1] to defray expenses. Radcliffe students are not eligible for membership."

The officers for 1917–18 are: President, Ralph Keffer Gr. (resigned at mid-years to enter war service), succeeded by John P. Ballantine '18; secretary and treasurer, Joseph L. Walsh Gr. (resigned at mid-years to enter war service), succeeded by D. S. Morse Gr. These officers, together with the faculty adviser, Dr. Gabriel M. Green, constitute the executive committee.

The following programs were given in 1917–18:

October 17: "History of Elementary Trigonometry" by Professor Maxime Bôcher;
November 7: "Curve Smoothing" by Lester R. Ford, instructor in actuarial mathematics;
November 21: "Solution of linear algebraic Equations in infinitely many Variables" by Joseph L. Walsh Gr.;
December 5: "On the Consistency and Equivalence of regular Transforma-

[1] The "small" fee is three dollars, an amount in excess of that charged at any of 40 other similar clubs in America.

tions" by Louis L. Silverman, instructor in mathematics at Cornell University;

December 19: "Rigid Motions in Space" by Professor Dunham Jackson;

January 9: "Some Methods of Interpolation" by John P. Ballantine '18;

February 20: "Finite Mathematics" by Henry M. Sheffer, instructor in philosophy.

THE MATHEMATICS CLUB OF HUNTER COLLEGE, New York City.

This club of young women was organized in 1910 "as the result of a desire on the part of both the teaching and student bodies to investigate matters connected with mathematics, to study the phases of mathematical development which are crowded out of classroom work, and to keep the students in touch with the best thoughts of the times. It aims to be a source of profitable pleasure." Motto: Honor habet onus.

All students "matriculated in the mathematics department" are eligible for membership, which now totals 80.

Officers 1917–18: President, Rose Sigal '18; vice-president, Dorothea Boves '19; secretary, Anita Rosenthal '19; treasurer, Miriam Werner, instructor in mathematics. Program committee: Professor Louisa M. Webster,[1] Rose Sigal '18, Kathryn McSorley '19, Mildred C. Zwinge '20, Jessie Krosovitch '21.

The programs for 1917 were as follows:

January 12: "The Slide Rule" by Adelheit Steeneck Gr.; "Origin of Geometrical Terms—a Sketch" written by Rose Sigal '18 and presented by 14 members of the club;

February 19: Reception to Freshmen;

March 9: "Mathematics of Physics" by Rosetta Chess '16; "Facts about Figures" by Jessie Krosovitch '21;

April 13: "Resources of Research" by Henryk Arctowski, chief of the science division, New York Public Library; "The Horn Book" by Grace H. Davis '20 (Illustrative photographs taken from Andrew Tuer's History of the Horn Book);

May 11: Guests of the Classical Club; "Three M's, Mysterious, Mystic, Magic" by Emory B. Lease, professor of Latin in the College of the City of New York; election of officers.

September 14: Reception to Freshmen;

October 19: "History of Trigonometry" by 13 Freshmen;

November 9: Report, by the club representative, of the meeting of the New York Division of the Association of Mathematics Teachers of the Middle States and Maryland; "The Mathematician at the Breakfast Table—a paper on mathematical Fallacies" by Mildred C. Zwinge '20;

December 13: "Personal Equation solved by judicial Process or Labor Problem

[1] Miss Webster's article on "Mathematics Clubs" in *The Mathematics Teacher* for June, 1917 (Vol. 9, pp. 203–208), contains information not given in these notes concerning the club of Hunter College.

solved" written by Kathryn McSorley '19 and presented by the Junior Class members. (The sketch was based on "*A*, *B* and *C*" in S. Leacock's *Literary Lapses*.)

In 1918 there was no meeting during January. At the one in February, twelve members showed the relation between mathematics and costuming; eight geometrically constructed gowns were on exhibition.

MATHEMATICAL CLUB OF ROCKFORD COLLEGE, Rockford, Ill.

So far as the editor knows this is the youngest of the undergraduate mathematics clubs, since it was organized only last October. It has 24 members.

Officers 1917–18: President, Estle Russell '18; vice-president, Dorothy Mandeville '20; secretary-treasurer, Aline Bartholomew '20.

The following programs have been given:

November 7: Opening address on the purpose of the club by the president;

November 21: "The History of Limits" by Marie S. Allen, instructor in mathematics;

December 21: A social meeting with readings from *Flatland, A Romance of Many Dimensions with Illustrations by the Author, A Square*;[1]

February 6: "Fourth Dimension" by Professor Bessie I. Miller, head of the department of mathematics.

THE JUNIOR MATHEMATICAL CLUB, University of Wisconsin, Madison, Wis.

As long ago as 1893 a Mathematics Club was organized at the University of Wisconsin for instructors, graduates, and seniors making mathematics their major. Its object was to follow important recent developments in mathematics. In March, 1912, the Junior Mathematical Club was founded and its meetings opened to all interested in mathematics. Most of the members, which now number 25, are Juniors and Seniors majoring in mathematics. The membership of the older organization, now called the "Senior Mathematical Club," is limited to members of the faculty and to graduate students.

Officers 1917–18: President, Florence Krieger '18; vice-president, Hilda Kieckhefer '19; secretary-treasurer, Kathryn Geiger '18. Program committee: Barbara Pearsall '19 (chairman), Frances McKay '18, Raymond Suchy '19.

Normally, meetings are held twice each month and are limited in length to one hour. The average attendance is about 18. In order that members may become better acquainted with each other and with members of the faculty during the year, there are a few social events such as picnics and hikes.

The following programs have been given in 1917–18:

[1] That is, A^2 = Abbott Abbott? Edwin Abbott Abbott was born in London in 1838 and he was still living there when the material for *Who's Who, 1918* was being collected. Educated at St. John's College, Cambridge, he graduated as senior in the classical tripos. He is the author of over 40 works, many of which are volumes of sermons or deal with the Gospels. Nearly a score of books including *A Shakespearean Grammar* (1870) and *How to Write Clearly* (1872) appeared before the first, and best, edition of *Flatland* (in small 4to) was published at London in 1884. This is the only mathematical work which Mr. Abbott has acknowledged as his.

November 7: Minutes; business; "General Survey of the History of Mathematics: (a) Greek, (b) Arabic, (c) Renaissance and Modern" by Professor Arnold Dresden; "Archimedes, His Life and His Work" by Albert Kohlman '18;

November 19: Minutes; business; "Eudoxus and his Method of Exhaustions which compares to our Method of Limits" by Professor Linnæus W. Dowling; "Euclid, his Life, Anecdotes, and a General Description of his Works" by Barbara Pearsall '19;

December 5: Minutes; business; "Pappus and his modern Elements as we find them in Greek" by Raymond Suchy '19; "Ptolemy and Greek Trigonometry" by Anne Clark '18; "What we forgot to mention concerning Mathematics of the Greeks" by Professor Edward B. Van Vleck; discussion;

January 23: Minutes; business; "What is a straight Line?"—two-minute discussions (including various definitions of a straight line) by every one present.

TOPICS FOR CLUB PROGRAMS.

Perusal of the recent very remarkable book by D'Arcy Wentworth Thompson, *On Growth and Form*,[1] has suggested to the editor that his next selection of topics be made from those which come up in the study of certain fields of botany and biology. The logarithmic spiral which bulks large in such discussion (pages 493–586 in Thompson's book) was chosen as one topic. While the others, Golden Section, and A Fibonacci Series, are less known to the average mathematician, there is much of interest, historical and otherwise, connected with their consideration.

8. The Logarithmic Spiral.[2]

The first discussion of this spiral, in a letter written by Descartes to Mersenne in 1638, was based upon the consideration of a curve cutting radii vectores, ρ (drawn from a certain fixed point, O), under a constant angle, ϕ. Descartes made the very remarkable discovery that if B and C are two points on the curve its length from O to B is to the radius vector OB as the length of the curve from O to C is to OC;[3] whence $s = a\rho$,[4] where s is the length measured along the curve from the pole to the point (ρ, θ), and $a = \sec \phi$.[5] This leads to the polar equa-

[1] Cambridge: at the University Press, 1917. 16 + 793 pp.

[2] Historical sketches and some of the properties of the curve are given in F. Gomes Teixeira, *Traité des courbes spéciales remarquables*, tome 2, Coïmbre, Imprimerie de l'université, 1909, pp. 76–86, 396–399, etc.; in G. Loria, *Spezielle algebraische und transzendente ebene Kurven*, Band 2, 2. Auflage, Leipzig, Teubner, 1911, pp. 60 ff.; in *Mathematisches Wörterbuch* . . . angefangen von G. S. Klügel . . . fortgesetzt von C. B. Mollweide, Leipzig, Band 4_1, 1823, pp. 429–440.

[3] *Oeuvres de Descartes*, tome 2, publiées par C. Adam et P. Tannery. Paris, Cerf, 1898, p. 360. *Cf.* I. Barrow, *Lectiones Geometricae*, Londini, 1670, p. 124; or English edition by J. M. Child, London, Open Court, 1916, pp. 136–9.

[4] The intrinsic equation $s^mR = K$ represents a logarithmic spiral when $m = -1$, a clothoïde when $m = 1$, a circle when $m = 0$, the involute of a circle when $m = -\frac{1}{2}$ and a straight line when $m = \infty$.

[5] That is, the length of the arc measured from the pole is equal to the length of the tangent drawn at the extremity of the arc and terminated by the line drawn perpendicular to the radius vector at the pole.

tion (1) $\rho = ke^{c\theta}$, where k is a constant and $c = \cot \phi$. The pole O is an asymptotic point. The pole and any two points on the spiral determine the curve; for, the bisector of the angle made by the radii vectores of the points is a mean proportional between the radii. If $c = 1$ the ratio of two radii vectores corresponds to a number, and the angle between them to its logarithm; whence the name of the curve.

The logarithmic spiral has been called also the proportional spiral[1] (E. Halley, 1696) but more commonly, because of the property observed by Descartes, the equiangular spiral (Whitworth, 1862). Johann Bernoulli employed the term loxodrome which is now reserved for the spherical curve which cuts all meridians under a constant angle. To Edmund Halley is usually ascribed the discovery that the loxodrome is the stereographic projection of a logarithmic spiral.[2]

Torricelli studied the logarithmic spiral about the same time that Descartes did. He gave a definition which may be immediately translated into equation (1), and from it he obtained expressions for areas and lengths of arcs. These results were rediscovered by John Wallis[3] and published in 1659.

During 1691–93 Jacob Bernoulli gave the following results among others: (*a*) Logarithmic spirals defined by equations (1) for different values of k are equal and have the same asymptotic point; (*b*) the evolute of a logarithmic spiral is another equal logarithmic spiral having the same asymptotic point;[4] (*c*) the pedal of a logarithmic spiral with respect to its pole is an equal logarithmic spiral;[5] (*d*) the caustics by reflection and refraction of a logarithmic spiral for rays emanating from the pole as a luminous point are equal logarithmic spirals.

The discovery of such "perpetual renascence" of the spiral delighted Bernoulli. "Warmed with the enthusiasm of genius he desired, in imitation of Archimedes,

[1] The lengths of segments cut off from a radius vector between successive whorls of the spiral form a geometric progression.

[2] *Philosophical Transactions*, 1696; but see two letters of Collins, one undated and the other dated Sept. 30, 1675, in *Correspondence of Scientific Men of the Seventeenth Century* . . . Vol. 1, Oxford, University Press, 1841, pp. 144, 218–19.

Cf. F. G. M., *Exercices de géométrie descriptive*, 4e éd., Paris, Mame, 1909, pp. 824–6. Chasles showed (*Aperçu historique*, etc., . . . 2e éd., Paris, 1875, p. 299) that if the logarithmic curve generates a surface by revolving about its asymptote, and if this asymptote is the axis of a helicoidal surface, the two surfaces cut in a skew curve whose orthogonal projection on a plane perpendicular to the asymptote is a logarithmic spiral. See also H. Molins, *Mémoires de l'académie des sciences inscriptions et belles-lettres de Toulouse*, tome 7 (sem. 2), 1885, p. 293 f.; tome 8, 1886, pp. 426. That the logarithmic spiral is a projection of a certain "elliptic logarithmic spiral" was shown in W. R. Hamilton, *Elements of Quaternions*, London, 1866, pp. 382–3. For other quaternion discussion of the logarithmic spiral see H. W. L. Hime, *The Outlines of Quaternions*, London, 1894, pp. 171–3.

[3] *Cf.* Turquan, "Démonstrations élémentaires de plusieurs propriétés de la spiral logarithmique," *Nouvelles Annales de Mathématiques*, tome 5, 1846, pp. 88–97.

[4] The center of curvature at a point on a logarithmic spiral is the extremity of the polar subnormal of the point.

[5] The nth positive pedal of the spiral $\rho = ke^{c\theta}$ with respect to the pole is

$$\rho = k \sin^n \phi e^{cn\left(\frac{\pi}{2}-\phi\right)} e^{c\theta}.$$

(J. Edwards, *Elementary Treatise on the Differential Calculus*, 3d edition, London, Macmillan, 1896, p. 167.)

to have the logarithmic spiral engraved on his tomb, and directed, in allusion to the sublime tenet of the resurrection of the body, this emphatic inscription be affixed—Eadem mutata resurgo." [1]

The logarithmic spiral appears in two propositions of Newton's *Principia* (1687).[2] From the first there develops that if the force of gravity had been inversely as the cube, instead of the square, of the distance, the planets would have all shot off from the sun in "diffusive logarithmic spirals." [3] In the second proposition Newton showed that the logarithmic spiral would also be described by a particle attracted to the pole by a force proportional to the square of the density of the medium in which it moves, while this density is at each point inversely proportional to its distance from the fixed center. This latter proposition was generalized by Jacob Bernoulli.

Cremona's discussion of the logarithmic spiral, and how it may serve, when drawn, for the solution of problems involving extraction of roots[4] (higher than the second) should not be forgotten. Then, too, there is the little known but notable paper, published by James Clerk Maxwell when only 18 years of age,[5] which contains several properties of logarithmic spirals. Some quotations follow:

Page 524 [10]: "The involute of the curve traced by the pole of a logarithmic spiral which rolls upon any curve is the curve traced by the pole of the same logarithmic spiral when rolled on the involute of the primary curve."

Page 529 [16]: "The method of finding the curve which must be rolled on a circle to trace a given curve is mentioned here because it generally leads to a double result, for the normal to the traced curve cuts the circle in two points, either of which may be a point in the rolled curve.

"Thus, if the traced curve be the involute of a circle concentric with the given

[1] *Cf. Acta eruditorum*, 1692, p. 212.

[2] Book I, proposition 9, and book II, proposition 15.

[3] The hodograph of an equiangular spiral is an equiangular spiral (W. Walton, *Collection of Problems in Illustration of the Principles of Theoretical Mechanics*, 3d ed., Cambridge, 1876, p. 296). In a chapter on electromagnetic observations in J. C. Maxwell's *Treatise on Electricity and Magnetism* (Vol. 2, Oxford, Clarendon Press, 1873, pp. 336–8) the discussion calls for the investigation of the motion of a body subject to an attraction varying as the distance and to a resistance varying as the velocity. This leads to the reproduction of Tait's application (*Proc. Royal Society of Edinburgh*, Vol. 6, 1869, p. 221 f.) of the principle of the hodograph to investigate this kind of motion by means of the logarithmic spiral.

"If a particle be describing a logarithmic spiral under the action of a force to the pole, and simultaneously the law of force be altered to the inverse biquadrate and the velocity to $\sqrt{\frac{2}{3}} \times$ its previous value, the particle will proceed to describe a cardioide." Purkis's *Messenger of Mathematics*, Vol. 2, 1864. For other results of this type, involving the spiral, see Newton's *Principia*, first book, Sections I–III, with notes and illustrations by P. Frost, London, 1880, p. 203.

[4] L. Cremona, *Graphical Statics*. Translated by T. H. Beare, Oxford, Clarendon Press, 1890, pp. 59–64. Italian edition, Torino, 1874, pp. 39–42. The xylonite logarithmic curve (eight inches in width) sold by Keuffel & Esser Co., New York, furnishes the means for accurately and rapidly drawing the curve. The curvature gradually changing it is peculiarly adapted for fitting to any part of a given curve. It assists in the rapid determination of the center of curvature of a given part of the curve, and, hence, in drawing evolutes and equidistant curves.

[5] "On the Theory of Rolling Curves," *Transactions of the Royal Society of Edinburgh*, Vol. 16, part V, 1849, pp. 519–40. [*The Scientific Papers of J. C. Maxwell*, edited by W. D. Niven, Vol. 1, Cambridge, 1890, pp. 4–29.] Loria, Gomes Teixeira, and Wieleitner seem to be equally ignorant of this paper.

circle, the rolled curve is one of two similar logarithmic spirals." (Often attributed to Haton de la Goupillière.)

Page 532 [19]: "If any curve be rolled on itself, and the operation repeated an infinite number of times, the resulting curve is the logarithmic spiral." The curve which being "rolled on itself traces itself is the logarithmic spiral."

Page 535 [23]: "When a logarithmic spiral rolls on a straight line the pole traces a straight line which cuts the first line at the same angle as the spiral cuts the radius vector." (Often attributed to Catalan.)

Among many other results the following may be noted: If a logarithmic spiral roll on a straight line the locus of the center of curvature of the point of contact is another straight line (A. Mannheim, 1859)—The involutes of a logarithmic spiral are equal spirals—The inverse of a logarithmic spiral with respect to its pole is an equal spiral with the same pole—Coplanar logarithmic spirals and their orthogonal trajectories, which are again coplanar logarithmic spirals, come up (1) in the discussion of loxodromic substitutions[1] and (2) in conformal representations.[2] As a consequence of a general theory relative to linear transformations F. Klein and S. Lie obtained the following result:[3] The logarithmic spiral is its own polar reciprocal with respect to each of the equilateral hyperbolas with center at the pole and tangent to the spiral.

The most practical form of a ship's anchor was discussed in 1796 by F. H. Chapman, vice-admiral in the Swedish Marine.[4] He found that the best form for each of the barbed arms would be an arc of a logarithmic spiral cutting the shank of the anchor at an angle of 67° 30'.

The first definite suggestion connecting the logarithmic spiral with organic spirals seems to have been made by Sir John Leslie in his *Geometrical Analysis and Geometry of Curve Lines*.[5] After proving that the involutes of a logarithmic spiral are logarithmic spirals he remarks: "The figure thus produced by a succession of coalescent arcs described from a series of interior centers exactly resembles the general form and the elegant *septa* of the *Nautilus*."[6] The aptness of this remark has been long since established. One of the earliest mathematical discussions of organic logarithmic spirals was by Canon Moseley, "On the Geometrical Forms of Turbinated and Discoid Shells"[7]—a paper of 80 years ago

[1] F. Klein–R. Fricke, *Vorlesungen über die Theorie der elliptischen Modulfunctionen*, Band I, Leipzig, Teubner, 1890, p. 168.

[2] G. Holzmüller, *Einführung in die Theorie der isogonalen Verwandtschaften und der conformen Abbildung*, Leipzig, Teubner, 1882, pp. 65, 238–241.

[3] *Mathematische Annalen*, Band 4, 1871, p. 77. *Cf. Encyklopädie der mathematischen Wissenschaften*, Band III_3, Leipzig, 1903, pp. 210, 212; also Clebsch-Lindemann, *Vorlesungen über Geometrie*, Band I, Leipzig, Teubner, 1876, p. 995.

[4] "Om rätta Formen på Skepps-Ankrar," *Svensk. Vetensk. Academ. nya Handl.*, 1796, Vol. 17, pp. 1–24. Abridged and translated in *Annalen der Physik* (Gilbert), Band 6, Halle, 1800: "Von der richtigen Form der Schiffsanker," pp. 81–95.

[5] Edinburgh, 1821, p. 438.

[6] For pictures of the nautilus pompilius see pp. 494, 581, 582 of Thompson's book and also T. A. Cook, *The Curves of Life*, London, Constable, 1914, pp. 57, 457. This latter work contains many beautiful illustrations and logarithmic spiral forms are specially discussed on pages 60–63, 413–421; another work by the same author, *Spirals in Nature and Art*, London, Murray, 1903, has some good illustrations.

[7] *Philosophical Transactions of the Royal Society*, London, 1838, Vol. 128, pp. 351–370.

which is one of the classics of natural history. In "turbinate" shells we are no longer dealing with a plane spiral as in the nautilus but with a gauche spiral on a right circular cone cutting the generators at a constant angle and such that along a generator the line-segments between successive whorls form a geometric progression.[1] For mathematical and other details of Moseley's work as well as of that of many others, on univalve and bivalve shells, Thompson's book, with its many exact references to the literature of the subject, should be consulted. One notable work which Thompson appears to have overlooked is Haton de la Goupillière, "Surfaces Nautiloïdes."[2]

In the field of leaf arrangement or phyllotaxis discussion of the theories of A. H. Church[3] and Cook evolved from observations of arrangements in logarithmic spirals of florets of sunflowers, pine cones, and other growths, should be read in connection with Thompson's criticisms. The fine sunflower photograph by H. Brocard[4] ought to be compared with those by Church.

NOTES AND NEWS.

EDITED BY D. A. ROTHROCK, Indiana University, Bloomington, Indiana.

Assistant Professor H. M. SHOWMAN, of Colorado School of Mines, has been promoted to a professorship of mechanics and engineering.

Assistant Professor S. T. SANDERS has been made head of the department of mathematics at Louisiana State University, and Dr. I. C. NICHOLS has been appointed associate professor of mathematics.

Assistant Professor L. P. SICELOFF, of the department of mathematics of Columbia University, has entered the Y. M. C. A. service, and will shortly embark for France.

Professor J. L. COOLIDGE, of Harvard University, has received a commission as major in the Ordnance Department of the National Army and is in active service. Dr. L. R. FORD, instructor in actuarial mathematics, has also entered the military service.

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[1] As early as 1701 Guido Grandi showed that the orthogonal projection of this spiral on a plane perpendicular to the axis of the cone is a logarithmic spiral. The gauche spiral has been studied by Th. Olivier (who called it the conical logarithmic spiral), *Developpements de géométrie descriptive*, 1843, pp. 56–76; by P. Serret, *Théorie nouvelle géométrique et mécanique des lignes à double courbure*, 1860, p. 101; etc. A number of results are collected by Gomes Teixiera, *l. c.*, pp. 396–400.

For other surfaces involving the logarithmic spirals reference should be given to the very interesting pages 232–313 of G. Holzmüller, *Elemente der Stereometrie*, Dritter Teil, Leipzig, Göschen, 1902, on logarithmic spiral tubular surfaces and their inverses.

[2] This occupies almost the whole of the third volume of *Annaes scientificos da academia polytechnica do Porto*, Coïmbra, 1908. *Cf. L'Intermédiaire des mathématiciens*, 1900, tome 7, p. 40; tome 8, pp. 167, 314; tome 17, p. 155.

[3] A. H. Church, *On the Relation of Phyllotaxis to Mechanical Law*, London, Williams and Norgate, 1904.

[4] In *L'Intermédiaire des mathématiciens*, 1909, and in H. A. Naber, *Das Theorem des Pythagoras*, Haarlem, Visser, 1908, opposite p. 80.